

K+#\$ INTRODUCTION

Arco is an analysis tool for masonry arches and vaults. The software is based on the “safe theorem” (also named “lower bound” or “equilibrium” theorem) of the plastic analysis method. The application of the ideas of plasticity theory to the study of masonry structures starts with Heyman’s works [Ref, 1]. The “safe theorem” can be stated as follow: a masonry arch is safe if a line of thrust, in equilibrium with external loads and lying wholly within the thickness, can be found and the corresponding stresses are sufficiently low.

The software includes the following features:

- segmental and user-definable intrados shape
- variable ring thickness
- backfill ‘passive’ pressures option
- automatic calculation of the ‘optimal’ thrust-line
- graphical output that can be exported in any CAD editor
- context sensitive help (press F1)

In the classical implementation of the equilibrium method the backfill is taken into account as a mere vertical surcharge, ignoring the horizontal passive pressures which can be mobilized when the arch sways into the backfill. This approach is generally on the safe side, but often does not allow to assess the stability of existing masonry arches, particularly in the case of asymmetrical live loads. The introduction of the horizontal pressures allows to assess the safety of arches and vaults also in cases in which the classical approach would fail (as in presence of sustained asymmetrical live loads). As a consequence, expensive and invasive rehabilitation interventions could be often avoided. From the point of view of restoration, best intervention is often no intervention.

Help is organized as a tutorial through three examples.

Example 1 is illustrated in detail, therefore should be tried first.

^KIntroduction

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[#] IDH_001

^{\$} Introduction

K+#\$ TUTORIAL - Example 1

File Help1.aro

Vault with intrados shape made of a single segment of circle.

Open file 'Help1.aro' using menu "File" or the button in the tool bar.

{bmc Bmp001.bmp} {bmc Bmp001-1.bmp}

Fig. 1 – Open file

Geometrical data are loaded from file.

{bmc Bmp002.bmp}

Fig. 2 – Geometry data

The meaning of each datum is illustrated in the sketch and explained as text moving the mouse on the relative box.

Click on the "**ZOOM**" button to enlarge the sketch (click again to reduce).

{bmc Bmp003.bmp}

Fig. 3 – Sketch enlarged

Distances d_w and d_h can be zero.

The ring width is irrelevant in this release since only distributed loads are considered. It influences only the value of the total load Q that is indicated in the plot.

Start eccentricities indicate the position of the hinges inserted (at springers and crown) to make the arch isostatic and to draw the thrust-line if no optimization is chosen in the "Solve options" form.

- Click on {bmc next.bmp} to plot the geometry.

{bmc Bmp004.bmp}

Fig. 4 – Geometry plot

The part of ring, through which the thrust-line will be drawn, is plotted in red.
On the bottom side the plan view of the extrados is plotted.

- Click on {bmc next.bmp} to input the live load.

K# Load

^K Example 1; Geometry (Example 1);Input (Example 1)

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[#] IDH_010

^{\$} Tutorial - Example 1

^K Load (Example 1)

[#] IDH_020

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Fig. 5 – Load input form

Following the ultimate limit states approach, partial factors for actions and resistances should be applied. Since the dead load effect is favourable as it tends to centre the thrust line, a partial factor value 0.9 was applied to the material densities. For the live load a partial factor value 2.0, greater than usual, may be used to allow also for uncertainties in the arch geometry.

In this example the vault has to be assessed for a sustained service load of 6 kN/m^2 : therefore a value $q=12 \text{ kN/m}^2$ is assumed. The load is applied to half span, condition that is generally the most severe for an arch.

- Click on {bmc next.bmp} to update the geometry plot.

{bmc Bmp006.bmp}

Fig. 6 – Geometry plot updated with the load

- Click on {bmc next.bmp} to input solve options.

K# Solve Options

{bmc Bmp007.bmp}

Fig. 7 – Solve options input form

Since the ‘Optimise thrust-line’ box is not checked, the thrust-line will pass through the three points identified by the ‘Start eccentricities’ in the ‘Geometry form’. See forward the options **“Optimize thrust-line”** and **“Passive pressures”**.

- Click on the **“Solve”** button.

K# Solution plot

{bmc Bmp008.bmp}

Fig. 8 – Solution plot

In the plot the value $Q=23,0 \text{ kN}$ (here highlighted in yellow) is the resultant of the distributed load q that weighs on the ring.

The trust-line is drawn in blue. To better represent the position of the thrust-line within the ring thickness, the arch profile is reproduced in the lower part of the drawing with the thickness magnified four times. The diagrams of the maximum stresses in each section at extrados (green curve) and intrados (magenta curve) are also plotted. Max stresses are calculated according to the classical elastic theory for no-tension materials, that is:

^K Solve Options (Example 1)

IDH_030

^K Solution plot (Example 1)

IDH_040

$$\sigma_{\max} = \frac{2N}{3u}$$

being N the compression axial force per ring unit width and u the distance of the trust-line from the compressed edge. If the trust-line lies outside the ring thickness, no equilibrium is possible without tension, and stresses are calculated as if the section were wholly reactive:

$$\sigma_{\max} = \frac{N}{t} \pm \frac{6Ne}{t^2}$$

Fig. 8 shows that the trust-line does not lay wholly within the ring thickness and therefore equilibrium would require a tension resistant masonry. The worst stress values at extrados ($\sigma_e \max = -1.15 \text{ Mpa}$) and at intrados ($\sigma_i \max = -0.623 \text{ Mpa}$) are both of traction.

- Click on the “**Report**” menu or button.

K# **Report form**

{bmc Bmp009.bmp}

Fig. 9 – Solution report form

- Click on the “**Enlarge**” button.

{bmc Bmp010.bmp}

Fig. 10 – Complete solution report form

In the report form, besides the σ values, also the percent of compressed section (0 if the thrust line lies outside thickness) is reported. Values are listed for each section in the grid (use the scroll bars to see all values). In the “Reactions at springings” frame the horizontal and vertical components of the thrust at left and right springings (Hl, Hr, Vl, Vr) and the trust in the tie (if present and positioned at springings level), are indicated. In the “Wall reactions” frame are indicated the horizontal resultant forces (Hl, Hr) and flexural moments (Ml, Mr) exerted on the lateral walls due to the passive pressures (if inserted).

K# **Optimize thrust-line**

Open the “**Data - Solve Options**” menu item or click the “**OPT**” button to improve the solution checking the “Optimize thrust-line” option box.

{bmc Bmp011-1.bmp} {bmc Bmp011-2.bmp}

{bmc Bmp011-3.bmp}

Fig. 11 – Optimize thrust-line frame

The ‘Steps n.’ value indicates the number of divisions in which the ring thickness is divided to vary the hinges position. The default value is 10 that represents a good compromise between accuracy

^K Report form (Example 1)

IDH_050

^K Optimize thrust-line (Example 1)

IDH_055

and solution time.

The 'Approx' value is the accuracy of the optimal hinges position and is initially equal to the maximum ring thickness divided by the steps number; it can be updated by the solver if in the first step (with hinges eccentricities set to zero) the thrust-line does not lay wholly within the ring thickness.

- Click on the “**Solve**” command or on the equivalent menu or button.

{bmc Bmp012.bmp}

Fig. 12 – Optimization procedure in progress

The optimization procedure can be stopped if it is too slow. In this case lower the step number and restart the solver. The solution is shown in the following figure:

{bmc Bmp013.bmp}

Fig. 13 – Solution plot after optimization

Now the thrust-line lies wholly within the ring thickness, but stresses are very large ($\sigma_e \text{ max}=9,33$ MPa, $\sigma_i \text{ max}=9,19$ MPa). The two values should be almost the same if the step number is sufficiently high. In the present case it is not worth increasing the accuracy; it is better to improve the solution introducing the fill passive pressure resistance.

K# Passive pressures

The arch, especially when not uniformly loaded, tends to deform toward the backfill, which reacts opposing its passive earth pressure resistance (Fig. 14).

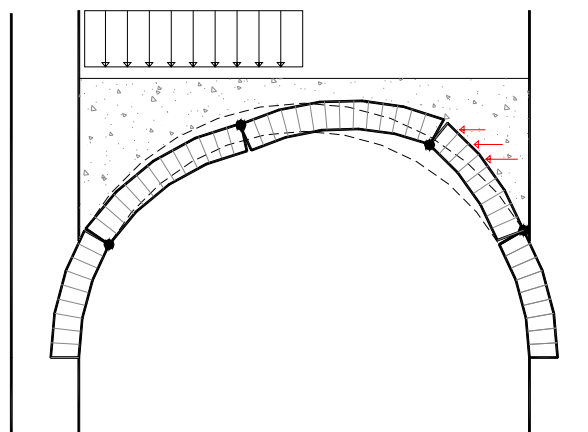


Fig. 14 – Failure mechanism counteracted by the fill horizontal passive pressures

On the voussoirs that tend to bend towards the fill the horizontal forces $S = k_P P_F \tan \alpha$ can be applied (Fig. 15).

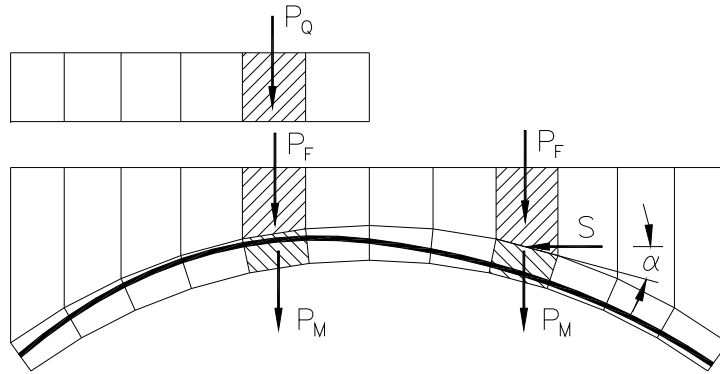


Fig. 15 – Horizontal force S, due to passive resistance, applied to the voussoir

A reasonable value for the passive pressure coefficient k_p , that would be achieved after small movements of the arch towards the fill, should be assumed. Values between 0.5 (coefficient at rest) and 2 are suggested. Let's try with $k_p=1$.

- Open the 'Solve options' form and check the 'Passive pressures' option box

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Fig. 16 – 'Passive pressures' frame

Choose the value $k_p=1$ for the passive pressure coefficient. Insert the number of voissiors subjected to passive pressures. Since in the present example the ring tends to deform toward the right side, introduce the number (16) of voissiors comprised between the crown and the right springer. Click on "Solve".

{bmc Bmp017.bmp}

Fig. 17 – Solution plot after optimization

Now the thrust-line lies wholly within the ring thickness and stresses ($\sigma_e \max=0,721$ MPa, $\sigma_i \max=0,714$ MPa) are sufficiently small. Slightly smaller values ($\sigma_e \max=0,655$ MPa, $\sigma_i \max=0,666$ MPa) can be obtained with $k_p=1,2$.

Note that the σ_{\max} value has to be compared with the masonry design resistance at ultimate limit state and not with its allowable stress. Moreover it is calculated considering an elastic distribution and reduces to 3/4 its value if a plastic distribution is assumed.

K# View menu

- Select the "View – Resultants" menu item to plot the forces vectors.

{bmc Bmp018.bmp}

Fig. 18 – Solution plot with resultant forces vectors

^K View menu (Example 1)

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Try the other view options.

K# Plot menu

The 'Plot' menu (Fig. 19) allows to print, to export in dxf format and to draw in Autocad (Rel. 14 or later).

{bmc Bmp100.bmp}

Fig. 19 –Plot menu

The introduction of the horizontal passive pressures acting on the ring implies the necessity to assess the equilibrium of the lateral walls (Fig. 20). The horizontal resultant force H_r and flexural moments M_r exerted on the right wall are indicated in the "Wall reactions" frame of the report form (Fig. 21).

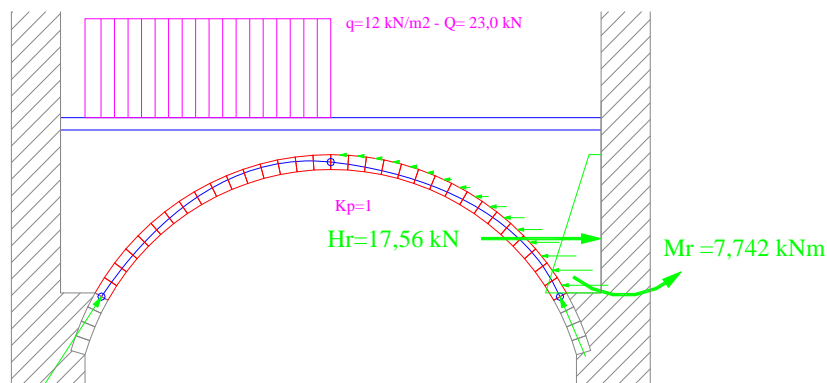


Fig. 20 –Actions on the lateral wall

{bmc Bmp110.bmp}

Fig. 21 –"Wall reactions" frame

In the "Optimal eccentricities" frame are indicated the eccentricities that optimize the thrust-line. Click on the "Set as start values" command if you want to use this values in subsequent analysis without using the optimization process.

{bmc Bmp120.bmp}

Fig. 21 –"Optimal eccentricities" frame

K+#\$ TUTORIAL - Example 2

File Help2.aro

Arch with user-defined intrados shape (lancet arch).

Open file 'Help2.aro' using menu "File" or the button in the tool bar as explained in [Example 1](#). Geometrical data are loaded from file. See [Example 1](#) for details.

{bmc Bmp201.bmp}

Fig. 1 – Geometry data of the lancet arch

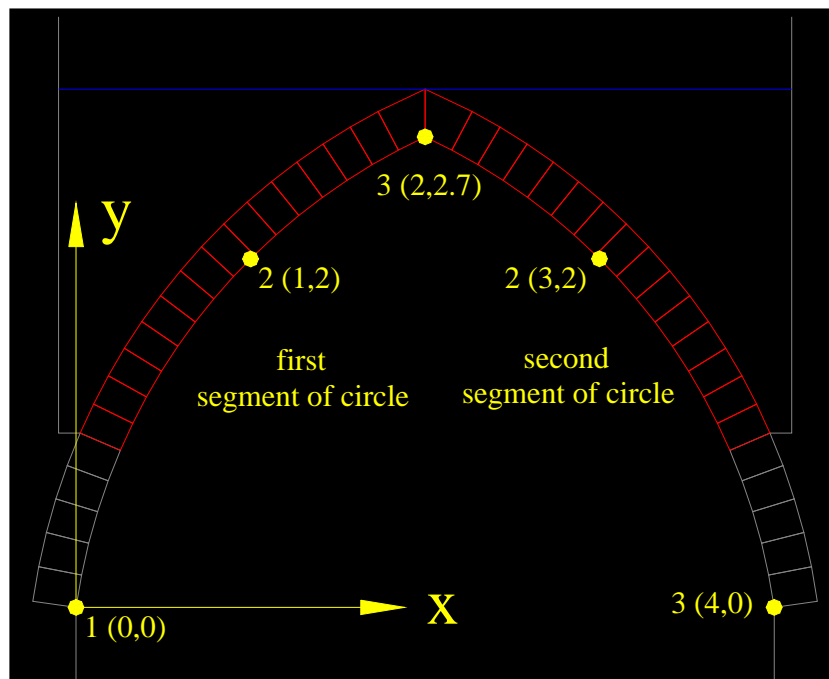


Fig. 2 – Input of the intrados shape

Fig. 2 explains the input of the intrados geometry. The user-defined arch shape is formed from a series of segments of circles. For each segment of circle co-ordinates of two points (points 2 and 3 in Fig. 2) should be given. The positions of all points should be measured relative to point 1, which has co-ordinates [0,0]. Subsequent points should be entered in order of increasing x distance.

- Click on {bmc next.bmp} to plot the geometry.

{bmc Bmp203.bmp}

Fig. 3 – Geometry plot

The two segments of arch are automatically connected.

^K Example 2; Input Example 2

⁺ automatic

[#] IDH_210

[§] Tutorial - Example 2

- Click on {bmc next.bmp} to input the live load.

{bmc Bmp204.bmp}

Fig. 4 – Load input form

See [Example 1](#) for details.

In this case the vault has to be assessed for a design load of 100 kN at crown. Since the arch ring width is 0,5 m, a value $q=400$ kN/m², applied from $x_{q1}=1,75$ m to $x_{q2}=2,25$ m, is assumed.

- Click on {bmc next.bmp} to update the geometry plot.

{bmc Bmp205.bmp}

Fig. 5 – Geometry plot updated with the load

- Click on {bmc next.bmp} to input solve options.

{bmc Bmp206.bmp}

Fig. 6 – Solve options input form

See [Example 1](#) for details.

- Click on the “**Solve**” button.

{bmc Bmp207.bmp}

Fig. 7 – Solution plot

See [Example 1](#) for details.

The thrust line-lies wholly within the ring thickness and stresses ($\sigma_e \max=1,39$ MPa, $\sigma_i \max=1,38$ MPa) are sufficiently small. Slightly smaller values ($\sigma_e \max=1,33$ MPa, $\sigma_i \max=1,33$ MPa) can be obtained with $k_p=2$.

K+#\$ TUTORIAL - Example 3

File Help3.aro

Barrel vault with polycentric intrados and variable thickness.

Open file 'Help3.aro' using menu "File" or the button in the tool bar as explained in [Example 1](#). Geometrical data are loaded from file. See [Example 1](#) for details.

{bmc Bmp301.bmp}

Fig. 1 – Geometry data of the barrel vault

The shape of the intrados (Fig. 2) is polycentric, and the radius of the central segment is given by the relation:

$$R = \frac{L \sin 60^\circ - f}{2 \cdot \sin 60^\circ - 1}$$

The arch has a 5m span and a 2m rise. The relatively high rise will enhance the influence of horizontal pressures. The ring depth is 12 cm (width of a brick) in the central segment and 24 cm (height of a brick) in the lateral segments towards the springings. Material design densities are 18 kN/m³ for the masonry and 16 kN/m³ for the fill, having applied a partial safety factor 0,9. The covering at crown is 20 cm. The design value of the live load is 12 kN/m², having applied a partial safety factor 2.

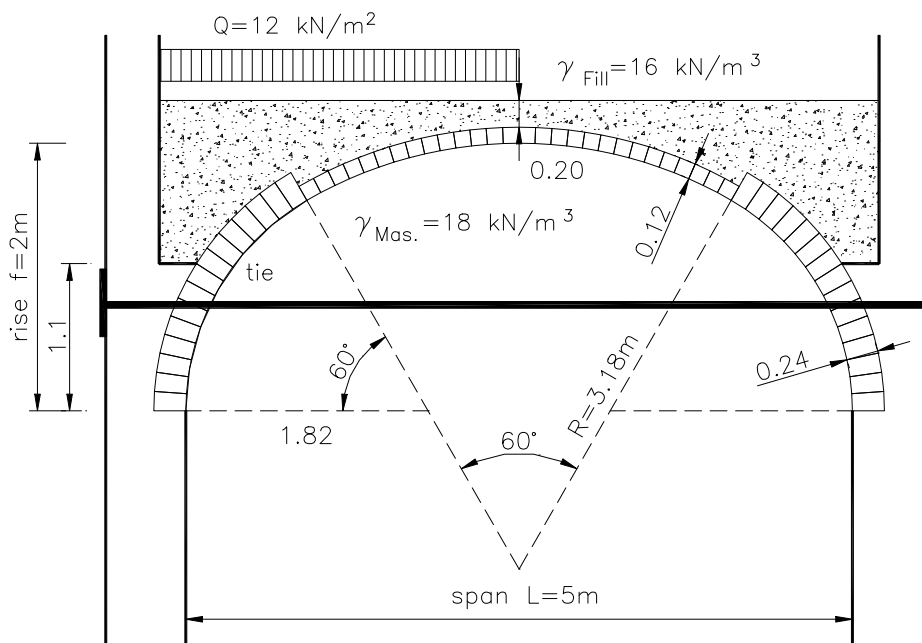


Figure 2: Geometry of the polycentric barrel vault.

^K Example 3; Input Example 3

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[#] IDH_310

^{\$} Tutorial - Example 3

The intrados shape is formed of 3 segments of circles; its geometry should be inserted as described in [Example 2](#) and shown in Fig. 3.

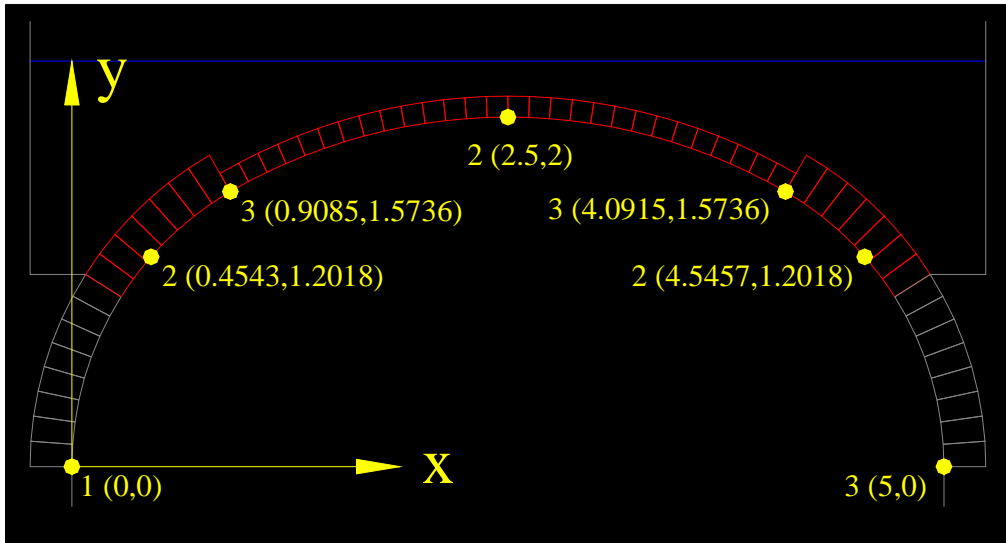


Fig. 3 – Input of the intrados shape

- Click on {bmc next.bmp} to plot the geometry.

{bmc Bmp304.bmp}

Fig. 4 – Geometry plot

- Click on {bmc next.bmp} to input the live load.

{bmc Bmp305.bmp}

Fig. 5 – Load input form

See [Example 1](#) for details.

- Click on {bmc next.bmp} to update the geometry plot.

{bmc Bmp306.bmp}

Fig. 6 – Geometry plot updated with the load

- Click on {bmc next.bmp} to input solve options.

{bmc Bmp307.bmp}

Fig. 7 – Solve options input form

See [Example 1](#) for details.

- Click on the “**Solve**” button.

{bmc Bmp308.bmp}

Fig. 8 – Solution plot with “View” menu items “Discretization” and “Horizontal forces” selected

See Example 1 for details.

The thrust-line lies wholly within the ring thickness and max stresses ($\sigma_e \max = 1,10 \text{ MPa}$, $\sigma_i \max = 1,09 \text{ MPa}$) are sufficiently small. The value $1,10 \text{ MPa}$, calculated considering an elastic distribution, reduces to 0.82 Mpa ($3/4 \times 1,10$) if a plastic distribution is assumed and can be accepted as design value also for a weak lime mortar.

K+#\$ **References**

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- [2] T. E Boothby, *Analysis of masonry arches and vaults*. Prog. Struct. Engng Mater. **3**, 246-256, (2001).
- [3] W. J. Harvey, *Application of the mechanism analysis to masonry arches*, The Structural Engineer **66**(5), 77-84, (1988).
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- [5] Gelfi, P., “Role of Horizontal Backfill Passive Pressure on the Stability of Masonry Vaults”, International Journal for Restoration of Buildings, Aedificatio Verlag, Freiburg, Vol. 8, No 6, pag. 573-589, ISSN 0947-4498, (2002).
<http://dicata.ing.unibs.it/gelfi/pdfPubblicazioni/14-Role of horizontal backfill.pdf>

^K References

⁺ automatic

[#] IDH_400

^{\$} References

K+#\$ Glossario

Per facilitare gli utenti italiani, viene riportata la traduzione di alcuni termini tecnici particolari.

Assessment = verifica

Backfill = rinfianco

Crown = chiave (dell'arco)

Safe theorem = teorema statico

Springer = primo concio alle reni

Springings = reni (dell'arco)

Thrust = spinta, tiro

Thrust-line = linea delle pressioni

Tie = catena

Voussoir = concio

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